FORMULARIO

MATEMATICA

REGOLA DEI SEGNI

$+ + \to +$ $\quad + - \to -$

$- - \to +$ $\quad - + \to -$

REGOLA DI CARTESIO

1) $>0;\ \geq 0$ $\quad x < x_1;\ x > x_2$ $\quad x \leq x_1;\ x \geq x_2$

2) $<0;\ \leq 0$ $\quad x_1 < x < x_2$ $\quad x_1 \leq x \leq x_2$

REGOLE FONDAMENTALI

$(a+b)^2 = a^2 + b^2 + 2a \cdot b$

$(a-b)^2 = a^2 + b^2 - 2a \cdot b$

$(a+b)^3 = a^3 + b^3 + 3a^2 b + 3a \cdot b^2$

$(a-b)^3 = a^3 - b^3 - 3a^2 b + 3ab^2$

$(a+b+c)^2 = a^2 + b^2 + c^2 + 2a \cdot b + 2a \cdot c + 2b \cdot c$

$a^2 - b^2 = (a-b)(a+b)$

$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

$a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

PROPRIETÀ RADICALI

$\sqrt[n]{a} \cdot \sqrt[n]{b} = \sqrt[n]{a \cdot b}$

$(\sqrt[n]{a})^m = \sqrt[n]{a^m}$

$\sqrt[n]{a^m} = a^{m/n}$

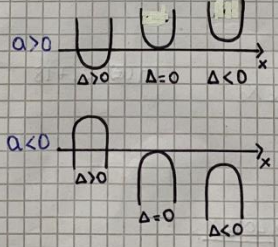
$\dfrac{\sqrt[n]{a}}{\sqrt[n]{b}} = \sqrt[n]{\dfrac{a}{b}}$

$\sqrt[n]{\sqrt[n]{a}} = \sqrt[n \cdot m]{a}$

EQUAZIONI 2° GRADO

$$x = \frac{-b \pm \sqrt{b^2 - 4a \cdot c}}{2 \cdot a}$$

$a(+) \cup$ $\quad a(-) \cap$



PROPRIETÀ DELLE POTENZE

$a^m \cdot a^n = a^{m+n}$

$a^m / a^n = a^{m-n}$

$a^m \cdot b^m = (ab)^m$

$a^m / b^m = (a/b)^m$

$(a^m)^n = a^{m \cdot n}$

$\sqrt[n]{a^m} = a^{m/n}$

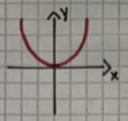
$a^0 = 1$

$a^{-n} = 1/a^n$

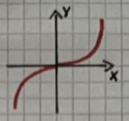
$(a/b)^n = \left(\frac{b}{a}\right)^{-n}$

FUNZIONI

Potenza con esponente

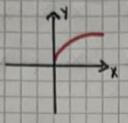


Pari

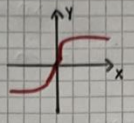


Dispari

Radice

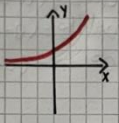


Pari

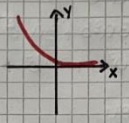


Dispari

Esponenziale

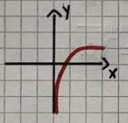


$a>1$

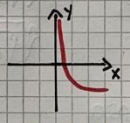


$0<a<1$

Logaritmica



$a>1$



$0<a<1$

LOGARITMI

$\log_a 1 = 0$

$\log_a (x_1 \cdot x_2) \Rightarrow \log_a x_1 + \log_a x_2$

$\log_a \frac{x_1}{x_2} \Rightarrow \log x_1 - \log x_2$

$\log_a (y^x) \Rightarrow x \cdot \log_a y$

$e = 2.718$

Cambio base $\Rightarrow \log_a x \Rightarrow \frac{\log_b x}{\log_b a}$

COEFFICENTE ANGOLARE RETTA

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

TRIGONOMETRIA

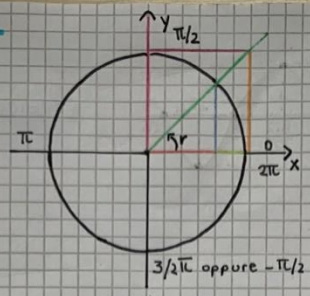
$$\frac{360°}{2\pi} = \frac{\alpha°}{\alpha_{rad}}$$

$$\alpha_{rad} = \frac{\pi}{180} \cdot \alpha°$$

$$\alpha_{°} = \frac{180}{\pi} \cdot \alpha_{rad}$$

$$\alpha = s/r$$

$$s = \alpha \cdot r$$



r
- = seno
- = senoverso
- = coseno
- = cosecante
- = tangente
- = cotangente
- = secante

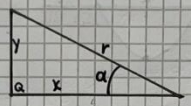
trigonometriche

$$\text{sen}(\alpha) = \frac{y}{r}$$

$$\cos(\alpha) = \frac{x}{r}$$

$$tg(\alpha) = \frac{TA}{AO}$$

triangolo rettangolo



$$\text{sen}\,\alpha = \frac{y}{r}$$

$$\cos\alpha = \frac{x}{r}$$

$$tg\,\alpha = \frac{y}{x}$$

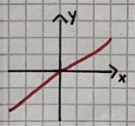
$$y = r\,\text{sen}\,\alpha$$

$$x = r\cos\alpha$$

$$y = x\,tg\,\alpha$$

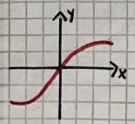
$\alpha°$	0	30	45	60	90
α_{rad}	0	$\pi/6$	$\pi/4$	$\pi/3$	$\pi/2$
sen	0	$1/2$	$1/\sqrt{2}$	$\sqrt{3}/2$	1
cos	1	$\sqrt{3}/2$	$1/\sqrt{2}$	$1/2$	0
tg	0	$1/\sqrt{3}$	1	$\sqrt{3}$	$+\infty$

funzioni trigonometriche inverse

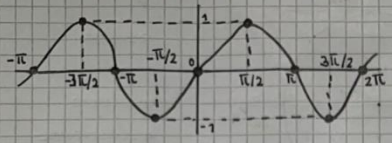


$y = \text{sen}\,x \rightarrow x = \text{arcsen}(y)\ /\ x = \text{sen}^{-1}(y)$

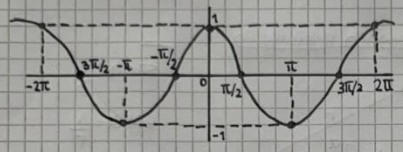
$x = \arccos(y)\ /\ x = \cos^{-1}(y)$



$x = \arctan(y)\ /\ x = tg^{-1}(y)$



funzione seno



funzione coseno

LIMITI

- infinito di $f(x)$ per $x \to a$ (n° finito) => ASINTOTO VERTICALE
- infinito di $f(x)$ per $x \to \infty$ => ASINTOTO ORIZZONTALE
- funzione continua => $\lim_{x \to c} f(x)$

forma indeterminata

- $\frac{\infty}{\infty}$ => quando $x \to \infty$ (dividi ogni numero per il massimo esponente)
- $0 \cdot \infty$ => quando $x \to a$ (n° finito) (raccogliere massimo esponente)
- $\frac{0}{0}$ => quando $x \to 0$ (sostituire con 0 le x)

DERIVATA PRIMA

- rapporto incrementale => $\frac{\Delta y}{\Delta x} = \frac{f(x_0+h) - f(x_0)}{h}$
- significato geometrico => $m_t = \lim_{h \to 0} (m_s) = \lim_{h \to 0} \left(\frac{\Delta y}{\Delta x}\right) = f'(x)$
- derivata di una funzione => $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

regole di derivazione

- Somma algebrica (2 o più funzioni) $y = [f(x) \pm g(x)]$ $y' = [f'(x) \pm g'(x)]$
- Prodotto (2 funzioni) $y = [f(x) \cdot g(x)]$ $y' = f'(x) \cdot g(x) + g'(x) \cdot f(x)$
- quoziente (2 funzioni) $y = \frac{f(x)}{g(x)}$ $y' = \frac{f'(x) \cdot g(x) - g'(x) \cdot f(x)}{[g(x)]^2}$

=>

- prodotto con k d(k) $f(x) = k \cdot d(x)$ (Costante)

(Se $f(x)$ è k allora)
d $y' = k \cdot g'(x)$

⇒ Derivazione funzioni elementari

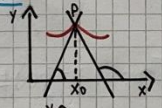
FUNZIONE	DERIVATA	FUNZIONE	DERIVATA
$y=c$	$y'=0$	$y=\operatorname{sen}x$	$y'=\cos x$
$y=x$	$y'=1$	$y=\cos x$	$y'=-\operatorname{sen}x$
$y=cx$	$y'=c$	$y=\operatorname{tg}x$	$y'=\frac{1}{\cos^2 x}$
$y=cf(x)$	$y'=c\,[f'(x)]$	$y=\log_a x$	$y'=\frac{1}{x}\log_a e$
$y=x^n$	$y'=n\,x^{n-1}$	$y=\ln x$	$y'=\frac{1}{x}$
$y=\sqrt{x}=x^{1/2}$	$y'=\frac{1}{2\sqrt{x}}=\frac{1}{2}x^{(1/2)-1}$	$y=a^x$	$y'=a^x\ln a$
$y=\sqrt[n]{x}=x^{1/n}$	$y'=\frac{1}{n\sqrt[n]{x^{n-1}}}=\frac{1}{n}x^{(1/n)-1}$	$y=e^x$	$y'=e^x$

derivate funzioni composte

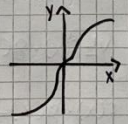
$F(x)=f[g(x)]$ $\quad F'(x)=f'[g(x)]\cdot g'(x)$

punti non derivabilità

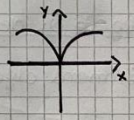
- spigolosità



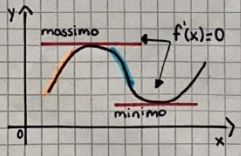
- tangente verticale



- cuspide



teorema di Fermat

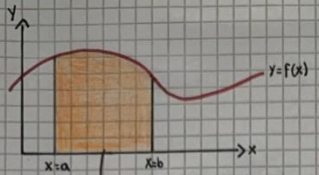


- derivata positiva
- derivata decrescente

Regola di de l'Hôpital

lim indeterminato ($\frac{0}{0}$; $\frac{\infty}{\infty}$) → sostituisco con derivate

INTEGRALI



Area $= \int_a^b f(x)\,dx$

$$\int_a^b f(x)\,dx = \int_a^b k\,dx = \overset{\text{funzione}}{k}(b-a)$$

$$\int_b^a f(x)\,dx = k(a+b) = -\int_a^b f(x)\,dx$$

$$\int_a^b f(x)\,dx = \int_a^b -k\,dx = -k(b-a)$$

Teorema

$$\int_a^b f(x)\,dx = F(b) - F(a) = \left[F(x)\right]_a^b$$

$f(x)$	$\int f(x)\,dx$
c	$cx+k$
x^n con $n \neq -1$	$\frac{x^{n+1}}{n+1}+k$
$1/x$	$\ln(x)+k$
e^x	e^x+k
$\sin(x)$	$-\cos(x)+k$
$\cos(x)$	$\sin(x)+k$

STATISTICA

Distribuzione frequenza

- numero classi => $2 \ln(N)$ (numerosità)
- campo variazione => $x_{max} - x_{min}$
- ampiezza classi => campo variazione / numero classi
- frequenza relativa => frequenza classe (f_i) / numerosità popolazione (N)
- frequenza assoluta => frequenza relativa (f_r) · numerosità (N)

$f_r \cdot 100 \rightarrow \%$

Media

- aritmetica $\bar{x} = (x_1 + x_2 + x_3 \cdots)/N$
- frequenza relativa $\bar{x} = 1/N \, \Sigma f_i \cdot x$ (frequenza ciascuna classe)
- ponderata $\bar{x} = \frac{\Sigma_i w_i \cdot x_i}{\Sigma_i \cdot w_i}$ (w_i peso)
- quadratica $RMS = \sqrt{\frac{1}{N} \Sigma x_i^2}$
- geometrica $G = \sqrt[N]{\Pi \cdot x_i}$
- armonica $H = \frac{N}{\Sigma \frac{1}{x_i}}$

RICORDA $H \leq G \leq \bar{x} \leq RMS$

Mediana N/2

- Riordinare dati ordine crescente
- Se dispari si prende elemento centrale al posto $(N+1)/2$
- Se pari media aritmetica numeri centrali corrispondono di posti $N/2$ ed $N/2+1$

Criterio equivalenza

Sostituire valore medio ad ogni valore → invariato → media utilizzata è giusta

Scostamento medio ($\bar{s}$)

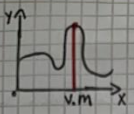
$\bar{s} = 1/N \cdot \Sigma s_i$ ($s_i = x - \bar{x}$ (scarto))

$\bar{s} = 0$ popolazione numeri uguali

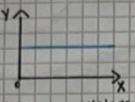
scostamento medio assoluto $MD = \frac{\Sigma_i |x_i - \bar{x}|}{N}$

La moda

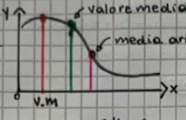
- valore modale



- No valore modale



- obliquo a sinistra



V.M < MEDIANA < MEDIA ARIT.

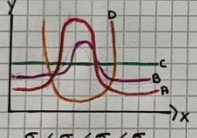
- obliquo a destra

MEDIA < MEDIANA < MODA

Differenza interquartile => $Q_3 - Q_1$

Varianza

$\sigma = \sqrt{\frac{1}{n}\Sigma(x_i - \bar{x})^2}$ (scarto quadratico medio) $S_i = (x_i - \bar{x})^2$ (Dati)



A = deviazione standard < induce dispersione basso valori concentrati

$\sigma_A < \sigma_B < \sigma_C < \sigma_D$

Coefficente variazione (C.V.)

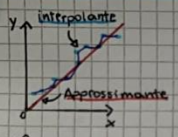
$C.V. = \frac{\sigma}{\bar{x}}$

Variabile standardizzata (Z)

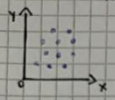
$Z = \frac{x - \bar{x}}{\sigma}$

- Varianza tot => $VT = \Sigma(y_i - \bar{y})^2/N$
- Varianza residua => $VR = \Sigma(y_i - y(x_i))^2/N$
- Cefficente di determinazione => $R^2 = (VT - VR)/VT = 1 - \frac{VR}{VT}$ (valori $R^2 = 0$ a 1)
- Coefficente di correlazione => $R = \sqrt{R^2} = \sqrt{1 - \frac{VR}{VT}}$
- Errore standard stima Y => $S_{XY} = \sqrt{VR}$

Approssimazione interpolazione ai minimi quadrati



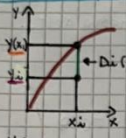
- passa per tutti i punti
- non passa per tutti i punti → Lineare (retta) / Non lineare (No retta)



Dati incorrelati

Curve → retta $Y = a_0 + a_1 \cdot X$
→ parabola $Y = a_0 + a_1 \cdot X + a_2 \cdot X^2$

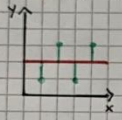
minimi quadrati (determina parametri modello)



$D_i = Y(x_i) - y_i$

Valore stimato

Valore osservato



$\Sigma(y_i - Y(x_i))^2$

$Y(x) = a_0$

$Y = a_0 + a_1 x$

$a_0 = (\Sigma y_i \cdot \Sigma x^2 - \Sigma x \cdot \Sigma x \cdot y)/(N \cdot \Sigma x^2 - (\Sigma x)^2)$

$a_1 = (N \cdot \Sigma \cdot x \cdot y - \Sigma x \cdot \Sigma y)/(N \cdot \Sigma x^2 - (\Sigma x)^2)$

Parabola

$D_i = y_i - Y(x_i) = y_i - a_0 \cdot x_i^2$ $\qquad E = \Sigma_i \cdot D_i$

$D_i^2 = y_i^2 - 2 y_i a_0 \cdot x_i^2 + a_0^2 \cdot x_i^4$

$E^2 = \Sigma_i \cdot D_i^2$

Pearson coefficiente di correlazione

$$r = \frac{\Sigma[(x_i + \bar{x})(Y(x_i) - \bar{Y})]}{\sqrt{\Sigma(x_i - x)^2 \cdot \Sigma(Y(x_i) - \bar{Y})^2}}$$

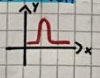
quando correlazione tra x e y è una retta

Probabilità

$\frac{h}{m} = n$ casi favorevoli / n casi possibili

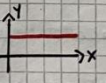
CURVE DI FREQUENZA

Campana (freq. =)



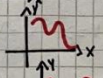
Ad U (-freq.)

Uniforme (dati uniformi)

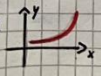
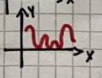


Discendente

Bimodale (no omogenea)



Ascendente



Pluri modale (sottogruppi omog.)